SOLUTIONS MAY 2021

ACTIVITIES FOR 1st AND 2nd ESO. 12-14 YEARS. AUTHOR: COLLECTIVE “CONCURSO DE PRIMAVERA”

(https://www.concursoprimavera.es/#concurso)

<u>May 1:</u> How many parts is a circle divided into if we draw on it 2021 different diameters?

<u>Solution:</u> Each diameter divides the circle into two parts. When adding a new diameter, each of the two component spokes divides a region in two. Therefore, 2021 diameters will divide the circle into (2 · 2021 =) 2042 parts.

<u>May 3:</u> As always different (equal) letters represent different (equal) digits

	A	**B**	**C**
	A	**B**	**C**
+	**A**	**B**	**C**
	B	**B**	**B**

<u>Solution:</u> What is required is equivalent to:

A	B	C
	x	3
B	B	B

By trial and error on the possible values of $C \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. We have that C = 0 is not possible, (because if C = 0 then B = 0 against B and C having different values). If we assume C = 1, then we should have

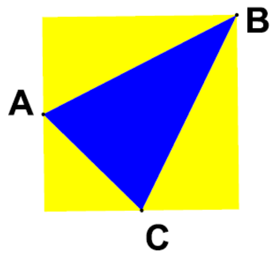
A	3	1
	x	3
3	3	3

which implies that the multiplication is wrong since 3 X 3 = 9. Following in this way we arrive at the possibility C = 8 in which case B = 4 and everything is stable (with A = 1)

1	4	8
	x	3
4	4	4

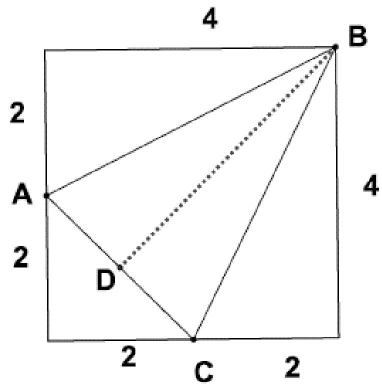
As the last possible value for C (9) also fails we have that the previous one is the only solution to the problem.

<u>May 4:</u> There is a square with a side 4m. If A and C are the midpoints, find the area of the triangle ΔABC



Solution: We have

$$\text{base} = \text{AC} = \sqrt{2^2 + 2^2} = 2\sqrt{2}$$
$$\text{AB} = \text{CB} = \sqrt{4^2 + 2^2} = 2\sqrt{5}$$



ΔABC is isosceles (AB = CD) and therefore, height = BD where D is the midpoint of AC.

$$\text{BD} = \sqrt{\text{BC}^2 - \text{DC}^2} = \sqrt{\left(2\sqrt{5}\right)^2 - \left(\sqrt{2}\right)^2} = 3\sqrt{2}$$

Finally:

$$\text{A}_{\Delta\text{ABC}} = \frac{\text{AC} \cdot \text{DB}}{2} = \frac{2\sqrt{2} \cdot 3\sqrt{2}}{2} = 6 \text{ m}^2$$

May 5: If we add the ages of three siblings in pairs, we get 26, 34 and 38 years. Calculate the age of the middle brother.

Solution: Let x, y, and z be the ages of the three siblings. From the statement, we have

$$\begin{aligned} x + y &= 26 \quad (1) \\ x + z &= 34 \quad (2) \\ y + z &= 38 \quad (3) \end{aligned}$$

In (1) the two youngest siblings participate, while (3) the two oldest siblings participate. Therefore, in (2) the brother who does not participate is the middle brother.

Adding the three equations we have:

$$2(x + y + z) = 98; \quad x + y + z = 49 \quad (4)$$

Finally, (4)–(1) leads to y = 15. The age of the middle brother is 15 years.

May 6-7: Dani walks up the stairs to her house two at a time and strides down three at a time. If he takes 7 more strides to go up than to go down, how many steps does the staircase to his house have?

Solution: Let x be the number of strides Dani takes when climbing. So x - 3 is the number of strides to go down. As the strides to go up are of two steps and the strides to go down are of three steps:

$$2x = 3(x - 7); \quad 2x = 3x - 21; \quad 21 = x$$

Then the number of steps is (2 · 21 =) 42 steps.

May 8: Five balls weigh the same as a spinning top and a yo-yo. A spinning top weighs the same as two balls and a yo-yo. How many balls weigh the same as two spinning tops?

Solution: If we represent by b the weight of a ball, by s the weight of a spinning top and by y the weight of a yo-yo, we can represent the information in the statement by:

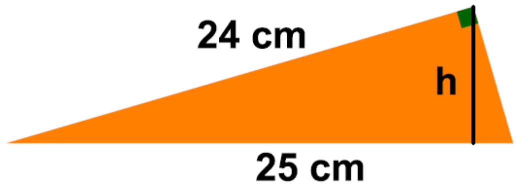
$$\begin{aligned} 5b &= 1s + 1y \\ 1s &= 2b + 1y \end{aligned}$$

Reversing the second equation and adding, we will have:

$$\left.\begin{aligned} 5b &= 1s + 1y \\ 2b + 1y &= 1s \end{aligned}\right\} \Rightarrow \quad 7b + 1y = 2s + 1y \quad \Rightarrow \quad 7b = 2s$$

That is, two spinning tops weigh the same as seven balls.

May 10-11: The hypotenuse of a right triangle is 25 cm and its leg is 24 cm, how high is the height that falls on the hypotenuse?



Solution: The other leg of the triangle will measure:

$$x = \sqrt{25^2 - 24^2} = 7$$

And now when calculating its area:

$$A = \begin{cases} = \dfrac{24 \cdot 7}{2} \\ = \dfrac{25 \cdot h}{2} \end{cases}$$

Therefore:

$$\frac{24 \cdot 7}{2} = \frac{25 \cdot h}{2} \Rightarrow 24 \cdot 7 = 25 \cdot h \Rightarrow \frac{24 \cdot 7}{25} = h = \frac{168}{25} \text{ cm} \approx 6{,}72 \text{ cm}$$

May 12: Aitana waters the plants on her terrace as follows: every day she waters the 12 pots or the 8 planters. If at the end of the week you watered 76 containers, how many days did you water the 12 pots?

Solution: Let x be the number of days that Aitana watered the 12 pots. So 7 – x, are the days that she watered the 8 planters. In addition, it must be met:

$$12x + 8(7 - x) = 76$$

Solving the equation, we have:

$$12x + 56 - 8x = 76; \quad 4x = 20; \quad x = 5$$

Then, Aitana watered the 12 pots that week for a total of 5 days.

May 13: The number 1a69b (where a and b are digits) is a multiple of 2, 9 and 11. Calculate a and b.

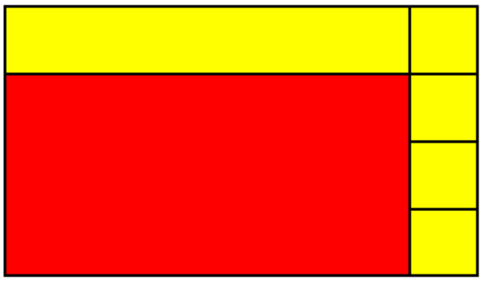
Solution: Remembering the divisibility criteria, we will have:

$$\begin{aligned} 1a69b = \dot{\hat{2}} \quad &\Leftrightarrow \quad b \in \{0, 2, 4, 6, 8\} \\ 1a69 = \dot{\hat{9}} \quad &\Leftrightarrow \quad 16 + a + b = \dot{\hat{9}} \\ 1a69 = \dot{\widehat{11}} \quad &\Leftrightarrow \quad 2 + a - b = \dot{\widehat{11}} \end{aligned}$$

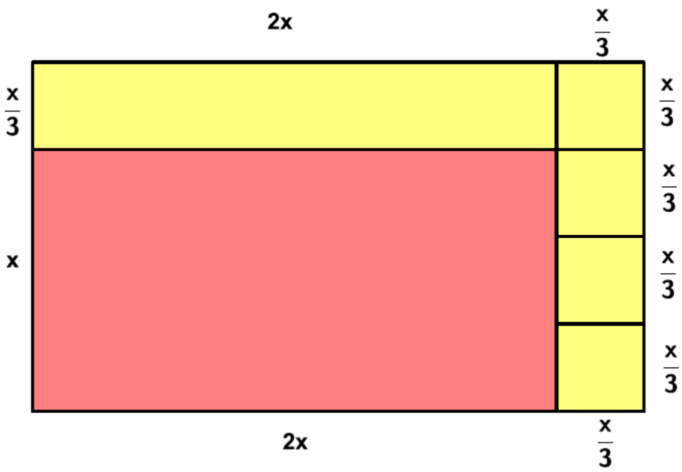
And now, calculating a for each of the possible cases of b and checking the condition of divisibility by 11

b	$16 + a + b = \hat{\dot{9}}$	¿$2 + a - b = \widehat{\dot{11}}$?	
0	a = 2	no	
2	a = 0	yes	10692
	a = 9	no	
4	a = 7	no	
6	a = 5	no	
8	a = 3	no	

May 14-15: To a red rectangle 54 cm in perimeter and with a base twice its height, four yellow squares and a rectangle have been added. What is the area of the rectangle formed by the six figures?



Solution: From the attached figure we will have:



Regarding the red rectangle:

$$6x = 54 \Rightarrow x = 9 \text{ cm}$$

Regarding the new rectangle:

$$\text{base} = 2x + \frac{x}{3} = \frac{7x}{3}$$

$$\text{height} = x + \frac{x}{3} = \frac{4x}{3}$$

Finally:

$$A = \frac{7x}{3} \cdot \frac{4x}{3} = \frac{28 \cdot x^2}{9} = \frac{28 \cdot 9^2}{9} = 252 \text{ cm}^2$$

May 17: The lengths of the sides of a rectangle are natural, the base is 7 cm longer than the height, and the sum of the lengths of three sides is 70 cm. Find the perimeter.

Solution: Suppose the rectangle has base b and height h. From the statement, we have: b = h +7. Since the sum of the lengths of three sides is 70, we will have:

$$\begin{cases} 2h + b = 2h + h + 7 = 70 \Rightarrow h = 21 \Rightarrow b = 21 + 7 = 28 \\ h + 2b = h + 2(h + 7) = 70 \Rightarrow 3h = 56 \Rightarrow h = \frac{56}{3} \notin \mathbb{N} \end{cases}$$

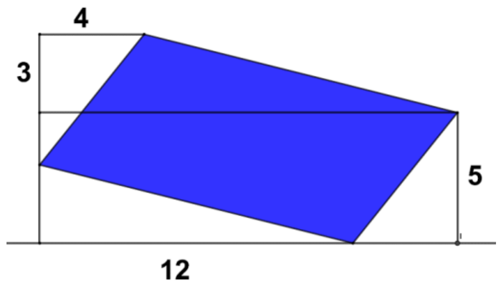
Therefore, only h = 21 and b = 28 are possible, in which case the perimeter is (2 · (21 + 28) =) 98 cm.

<u>May 18-19:</u> Aitana observes in the laboratory how bacteria reproduce. On the first day there were 1000, the second twice as many as the first, the third three times as many as the second, the fourth there were four times what there was on the third. How many bacteria would you say there will be on the tenth day?

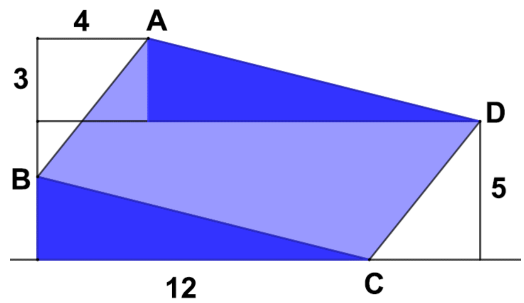
<u>Solution:</u> If b_i indicates the number of bacteria on day i, we will have:

$$\begin{aligned} b_{10} &= 10\cdot b_9 = 10\cdot 9\cdot b_8 = 10\cdot 9\cdot 8\cdot b_7 = 10\cdot 9\cdot 8\cdot 7\cdot b_6 = 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot b_5 \\ &= 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot b_4 = 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot b_3 = 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot b_2 \\ &= 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot b_1 = 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1000 = 3{,}628{,}800 \end{aligned}$$

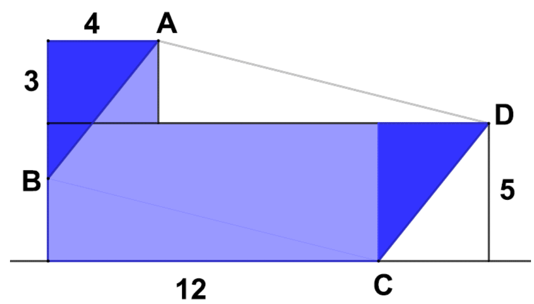
<u>May 20:</u> Find the area of the blue parallelogram



<u>Solution:</u> We slide the highlighted triangle of the blue parallelogram along the edges AB and DC

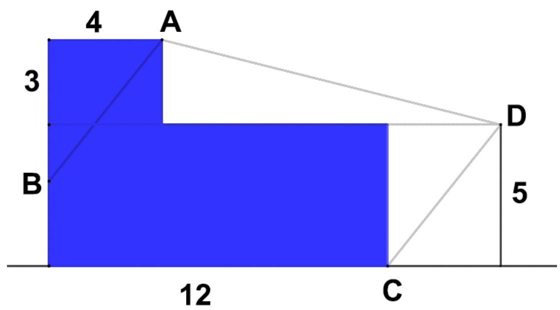


In the same way, we translate the highlighted triangle of the blue parallelogram along the DA and CB edges.

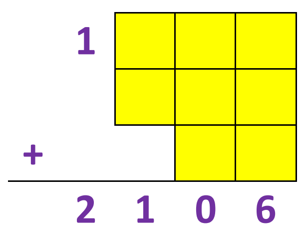


It remains for us to calculate the area of the blue zone. But this calculation is simple: It consists of two rectangles, one of base 12 and height 5 and the other of base 4 and height 3.

$$A = 3\cdot 4 + 12\cdot 5 = 72$$



<u>May 21:</u> Complete with the figures from 2 to 9, without repeating any, the yellow cells so that the sum is correct



Solution: The smallest possible value (for ones and tens) is (2 + 3 + 4 =) 9 and the largest value is (9 + 8 + 7 =) 24. Therefore, necessarily, the sum of the ones must be 16 and the sum of the tens must be 19 (so that with the one that we carry we obtain 20) and the sum of the hundreds must be 9 (so that with the two that we carry of the tens, we obtain 11) and thus the sum will be well done. Thus, we must partition the set {2, 3, 4, 5, 6, 7, 8, 9} into three subsets, two subsets of three elements with sums of elements 19 and 16 and the other subset of two elements with sum of elements 9.

The only possibilities for the hundreds are:

(1) 9 = 7 + 2	(2) 9 = 6 + 3	(3) 9 = 5 + 4

For case (1), we have:

19 = 9 + 6 + 4	19 = 8 + 6 + 5
16 = 8 + 5 + 3	16 = 9 + 4 + 3

For case (2), we have:

19 = 9 + 8 + 2	19 = 8 + 7 + 4
16 = 7 + 5 + 4	16 = 9 + 5 + 2

For case (3), we have:

19 = 9 + 8 + 2	19 = 9 + 7 + 3
16 = 7 + 6 + 3	16 = 8 + 6 + 2

The solutions are:

	1	7	9	8
		2	6	5
+			4	3
	2	1	0	6

	1	7	8	9
		2	6	4
+			5	3
	2	1	0	6

	1	6	9	7
		3	8	5
+			2	4
	2	1	0	6

	1	6	8	9
		3	7	5
+			4	2
	2	1	0	6

	1	5	9	7
		4	8	6
+			2	3
	2	1	0	6

	1	5	9	8
		4	7	6
+			3	2
	2	1	0	6

Each of these solutions can interchange the figures in each column. Therefore, each of these solutions actually carries (3! · 3! · 2! =) 72 solutions

May 22: A week ago 10% of Laia's class had the flu. Today, 10% of the sick were cured and 10% of the healthy got sick. What percentage of the class has the flu?

Solution: Let x be the size of the class. A week ago we had:

$$10\% \text{ of x had the flu} \Rightarrow \begin{cases} \frac{10x}{100} = \frac{x}{10} \text{ had the flu} \\ \frac{9x}{10} \text{ had no flu} \end{cases}$$

This week we will have:

$$90\% \text{ of those who had the flu still have the flu} \Rightarrow \text{with flu for two weeks} = \frac{90}{100} \cdot \frac{x}{10} = \frac{9x}{100}$$

$$10\% \text{ of those who did not have the flu have the flu} \Rightarrow \text{with flu only this week} = \frac{10}{100} \cdot \frac{9x}{10} = \frac{9x}{100}$$

Therefore:

$$\text{with flu the second week} = \frac{9x}{100} + \frac{9x}{100} = \frac{18x}{100} = 18\% \text{ of x}$$

May 24/31-25: Complete the attached diagram, knowing that it is an addition table (that is, a + b = 8, and so on all the other boxes), that the largest number that appears is 21, and that all 15 natural ones are different

+	b		
a	8	12	
	10		
	13		

Solution: Since there is a difference of 2 between 8 and 10, there must also be a difference between the second and third entries in the first column. Since there is a difference of 5 between 8 and 13, there must also be a difference between the second and fourth entries in the first column. Similarly between 12 and 8 there is a difference of 4 that must also be kept between 10 and the third entry in the third column and between 13 and the fourth entry in the third column and between b and the first entry in the third column

+	b	b+4	
a	8	12	
a+2	10	14	
a+5	13	17	

Since the highest number that appears in the table is 21, this value must be in the fourth entry of the fourth column. And since there is a difference of 4 between 17 and 21, this difference must be maintained between the entries in the fourth and third column

+	b	b+4	b+8
a	8	12	16
a+2	10	14	18
a+5	13	17	21

It remains to fill the first row and column with the requirement that there are 15 different naturals. That is, a and b must be chosen from {1, 2, 3, 4, 5, 6, 7, 9, 11, 15, 16, 19, 20} so that a + b = 8 and also a ≠ b are satisfied . That is: a = 1 and b = 7 or a = 2 and b = 6 or a = 3 and b = 5. In short, the possible solutions are: (and their transpositions between first row and column)

+	7	11	15
1	8	12	16
3	10	14	18
6	13	17	21

+	6	10	14
2	8	12	16
4	10	14	18
7	13	17	21

+	5	9	13
3	8	12	16
5	10	14	18
8	13	17	21

May 26: The guests at a wedding occupied several tables of 7 people. As they were very tight, 3 more tables were prepared and then all the tables were occupied, but with 6 people per table. How many guests were there?

Solution: If x is the number of tables with 7 people, x + 3 is the number of tables with 6 people. Therefore:

$$7x = 6(x + 3) \quad \Rightarrow \quad x = 18$$

Then at the wedding there were (7·18 =) 126 guests.

May 27: As always different (equal) letters represent different (equal) digits

$(LEE)^2$ = PEDAL

Solution: As the highest (lowest) possible value for PEDAL is 98765 (10234) we will have:

$$101 < \sqrt{10234} \leq LEE = \sqrt{PEDAL} \leq \sqrt{98765} < 315$$

Therefore, L can only take the values 1, 2 or 3.

If we assume L = 1, as $(1EE)^2 = PEDA1$, ends in 1 (= L), we look for a figure E such that

$$E^2 = 10x + 1$$

The only possibility (excluding L = 1 = E, to go against the statement) is E = 9 (see attached table). In this case:

$$(LEE)^2 = (199)^2 = 30601 = PEDAL$$

And therefore, all the requirements of the statement are met. We already have a solution.

If we assume L = 2, as $(2EE)^2 = PEDA2$, ends in 2 (= L), we look for a figure E such that

$$E^2 = 10x + 2$$

But, according to the attached table for no possible value of E its square ends in 2. That is, L = 2, it does not provide any solution.

If we assume L = 3, as $(3EE)^2 = PEDA3$, ends in 3 (= L), we look for a figure E such that

$$E^2 = 10x + 3$$

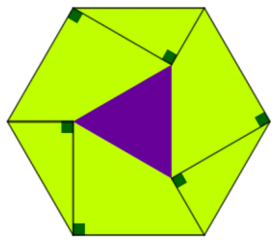
But, according to the attached table for any possible value of E, its square ends in 3. That is, L = 3, it does not provide any solution:

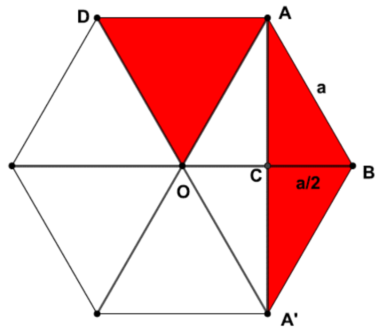
E	E^2
0	0
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81

The only solution to the problem posed is:

$$(LEE)^2 = (199)^2 = 30601 = PEDAL$$

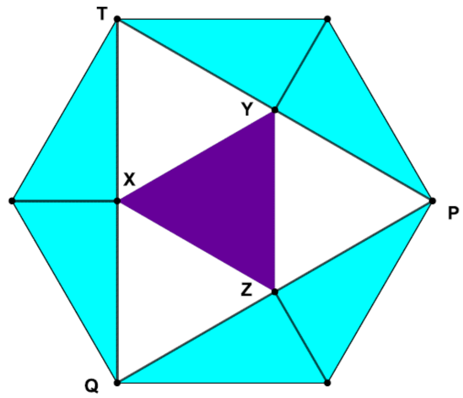
May 28-29: With the help of some perpendiculars we have drawn a purple triangle inside the green regular hexagon. If the area of the regular hexagon is 120 cm^2, what is the area, in cm^2, of the purple triangle?





Solution: Recall that a regular hexagon with side a, we can consider it to be composed of six equilateral triangles with side a. If that is the case, we will have:

$$A_{\Delta OAD} = A_{\Delta OAB} = 2 \cdot A_{\Delta ACB} = A_{\Delta ACB} + A_{\Delta CBA'} = A_{\Delta AA'B}$$



Based on the above, in the accompanying illustration, the area of the blue triangles equals half the area of the hexagon. From here:

$$A_{\Delta PTQ} = \frac{120}{2} = 60$$

Since the triangle ΔXYZ divides into four triangles equal to the triangle ΔTPQ, we will have:

$$60 = A_{\Delta TPQ} = 4 \cdot A_{\Delta XYZ} \;\Rightarrow\; \frac{60}{4} = A_{\Delta XYZ} = 15\ \text{cm}^2$$