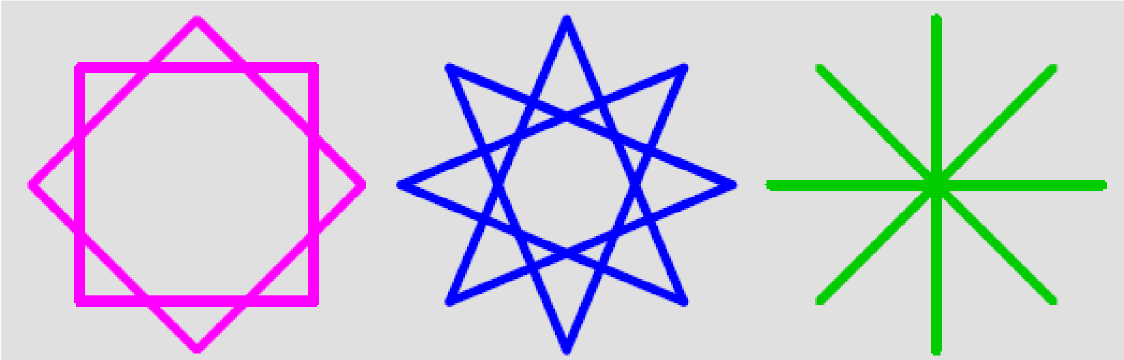
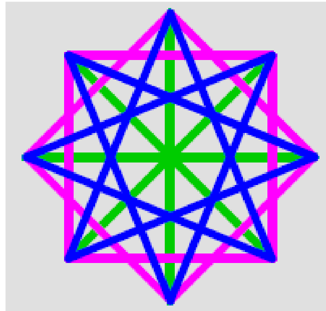
OCTOBER

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	SUN.





1

2

1.- If $x=\left(1+\frac{1}{n}\right)^{n}$ and $y=\left(1+\frac{1}{n}\right)^{n+1}$ compare x^{y} whit y^{x}

2.- Prove that, for every positive integer n:

$1^2-2^2+3^2-4^2+\cdots+(-1)^{n}\cdot(n-1)^2+(-1)^{n+1}\cdot n^2=(-1)^{n+1}\cdot(1+2+\cdots+n)$

3

4 **5**

Let n be a fixed positive integer. To any choice of n real numbers that satisfy $0\le x_i\le 1$ ($i\in\{1, 2, .., n\}$) we make them correspond the sum

$$\sum_{1\le i<j\le n}|x_i-x_j|=|x_1-x_2|+|x_1-x_3|+\cdots+|x_1-x_n|+|x_2-x_3|+\ldots+|x_2-x_n|+\ldots+|x_{n-1}-x_n|$$

Find the largest possible value of this sum

6

Simplify:

$$\sqrt[3]{\frac{1\cdot2\cdot4+2\cdot4\cdot8+\cdots+n\cdot2n\cdot4n}{1\cdot3\cdot9+2\cdot6\cdot18+\cdots+n\cdot3n\cdot9n}}$$

7

8

A function y = f (x) is said to be periodic if there exists a positive real number p such that f (x + p) = f (x) for all x. For example, y = sin x is periodic of period 2π. Is the function:

$y=\sin(x^2)$

periodic? Prove your claim

9

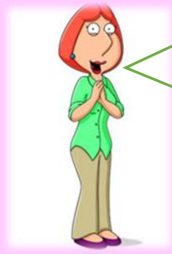
A sequence $a_1, a_2, a_3, \ldots$ satisfies that $a_1 = ½$ and $a_1+a_2+\cdots+a_n=n^2\cdot a_n$, for any n. Determine the value of a_n

10

11

Let A, B, C and D be four consecutive points of a circle and P, Q, R and S the midpoints of the arcs AB, BC, CD and DA. Prove that PR ⊥ QS

12



13

For each real r, we define:

$[r]=\max\{z\in\mathbb{Z}\mid z\le r\}$

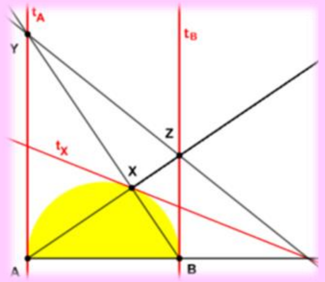
(e. g. [6] = 6; [π] = 3; [-1,5] = -2)). Draw the set of points on the (x, y) plane:

$[x]^2+[y]^2=4$

14

Given four weights that are in arithmetic progression and a two-arm scale, show how to find the largest weight using the scale only twice.

15



16

Given a circle of diameter AB and a point X different from A and B, let t_A, t_B and t_X be the tangents to the circle at A, B and X. Let Z be the intersection of AX with t_B and Y the intersection of BX with t_A. Prove that the three lines AB, t_X and ZY are concurrent or parallel

17

18 **19**

We have an unlimited number of 8-cent and 15-cent stamps. Some amounts of postage cannot be obtained exactly, e. g. 7 cents or 29 cents. What is the largest quantity that cannot be obtained exactly, i. e. the amount of postage that cannot be reached exactly, while all higher amounts are achievable?

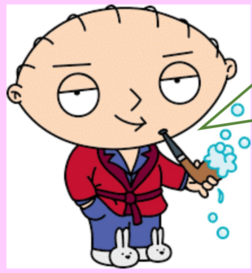
20

Let AB be one of its diameters of a given circle. Let C be a fixed point on segment AB and Q be a variable point on the circumference of the circle. Let P be a point on the line determined by Q and C for which:

$$AC/CB = QC/CP$$

Describe the locus of point P

21



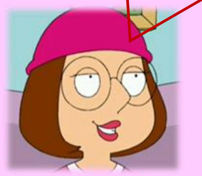
22

Show that if a number is rational, the decimal part, the integer part, and the number cannot be in geometric progression.

Find a positive number such that its decimal part, its integer part and the number are in geometric progression

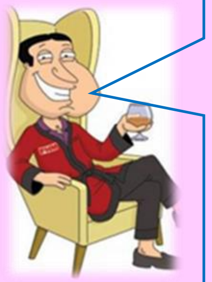
23

Let n be a positive integer. Prove that if n is a power of 2 then n cannot be put as the sum of consecutive positive integers



24

25 **26**



Suppose:

$n\cdot(n+1)\cdot a_{n+1}=n\cdot(n-1)\cdot a_n-(n-2)\cdot a_{n-1}$

for every positive integer $n\ge 1$. Si $a_0 = 1$ and $a_1 = 2$, find:

$$\sum_{i=0}^{50}\frac{a_i}{a_{i+1}}=\frac{a_0}{a_1}+\frac{a_1}{a_2}+\cdots+\frac{a_{50}}{a_{51}}$$

27

28

Let ABCD be a rectangle with BC = 3 · AB. Prove that if P and Q are points on BC with BP = PQ = QC, then:

∠DBC + ∠DPC = ∠DQC

29

30

Two seventh grade students were allowed to play in an eighth grade chess tournament. Each pair of participants played each other once and each of the participants received one point for winning each game, half for drawing a draw, and zero points for each game lost. The two seventh graders received a total of eight points, and the eighth graders all earned the same number of points. How many eighth grade students participated in the tournament? Is it the only solution?

31