MATHEMATICAL CALENDAR 2022-2023

Societat d'Educació Matemàtica Al-Khwarizmi

The Mathematical Education Society of the Valencian Community Al-Khwarizmi is a society of teachers of Mathematics. The objectives of the Company are, in accordance with its statutes:

1. Disseminate mathematics and the various currents of mathematical thought.
2. Transmit educational innovations in the teaching and learning of mathematics.
3. Promote the development and dissemination of research in Mathematics Education.
4. Encourage all those activities aimed at overcoming obstacles to the dissemination of mathematics generated by cultural or gender reasons.
5. Collaborate and exchange information with Associations and Societies of a similar nature and purpose.
6. Collaborate with institutions and entities to carry out studies and activities related to Mathematics and Mathematics Education.
7. Carry out studies, critiques and curricular proposals for any of the educational levels.

If you consider that these objectives are important, please contact us on the page

http://www.semcv.org/.

DIPUTACIÓ DE CASTELLÓ

ACTIVITY RESOLUTION CONTEST

ANNOUNCEMENT

1. **TO THE MOST INGENIOUS SOLUTION: Any ESO, Baccalaureate or F. P. student who gives an answer (solution/comment) to an activity proposed on any given day can participate. Each center will select the best solutions from their students by sending only one for each day and including: full name of the student, course and level, center, address, telephone and email. The winners will receive the corresponding accrediting diploma.**

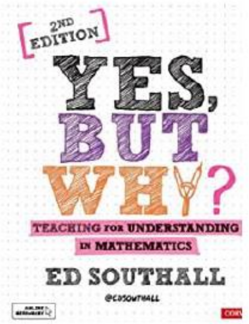
2. **TO GROUP WORK: A single group from any ESO and/or Baccalaureate and/or Vocational Training center that responds (solution/comment) to all the activities proposed in any given month may participate. The full name of the center, address, telephone and email must be indicated, as well as the name of all the students that make it up and the professor who coordinates it. The winners will receive the corresponding accrediting diploma.**

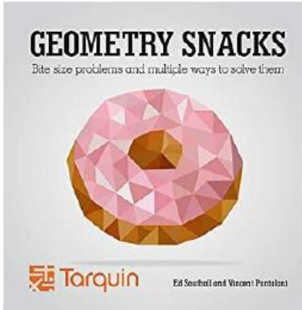
3. **PRESENTATION AND SELECTION: The reception period will end on the last day of the following month to which the activities correspond. Solutions should be directed to calendari@semcv.org**
   The solutions presented can be published when the selection committee deems it appropriate.

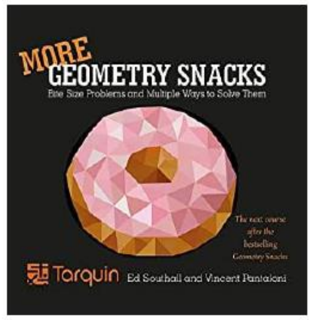
calendari@semcv.org

SEPTEMBER

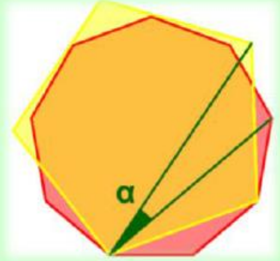
MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	U



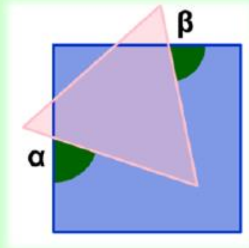




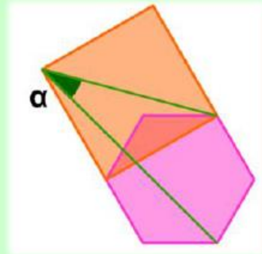
1 Pentagon and enneagon, regulars. Find reasonably α



2 Square and equilateral triangle. Find reasonably α + β



3 Square and regular hexagon. Find reasonably α



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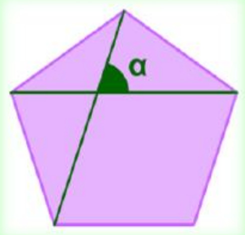
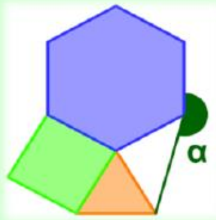
5 Regular pentagon. Reasonably find α



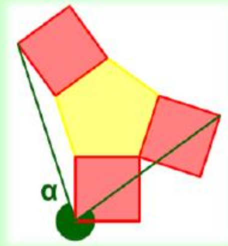
6 Equilateral triangle, square and regular hexagon. Find reasonably α



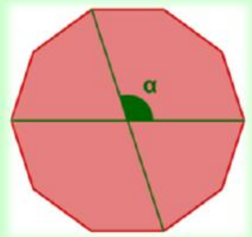
7 Regular octagon and equilateral triangle. Find reasonably α

8 Square and regular octagon. Find reasonably α

9 Regular pentagon and three squares. Find reasonably α



10 Regular decagon. Find reasonably α



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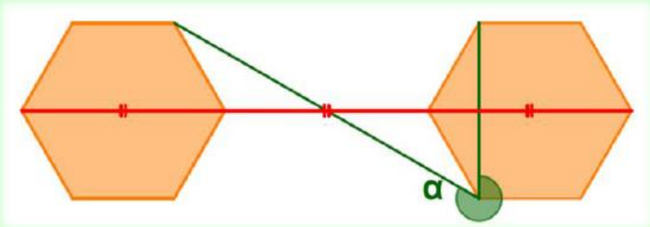
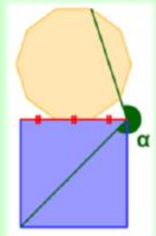
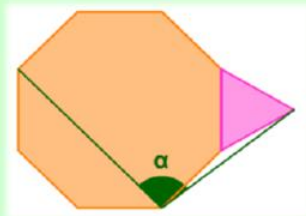
12 Regular hexagons **13** Find reasonably α



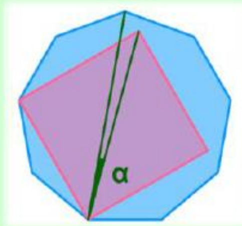
14 Regular and square decagon. Find reasonably α



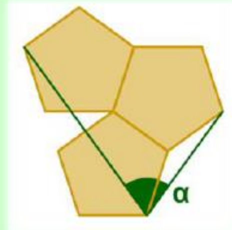
15 Regular octagon and equilateral triangle. Find reasonably α



16 Square and regular enneagon. Find reasonably α



17 Three regular pentagons. Find reasonably α



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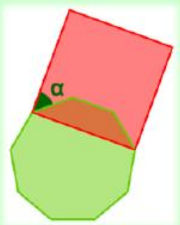
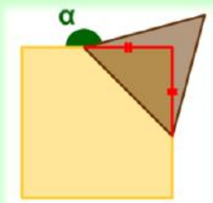
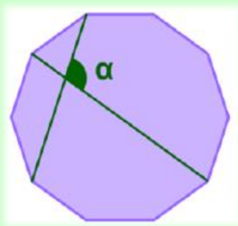
19 Square and regular enneagon. Find reasonably α



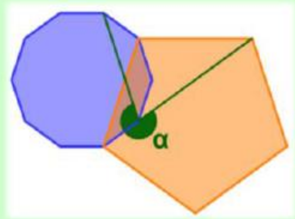
20 Square and equilateral triangle. Find reasonably α



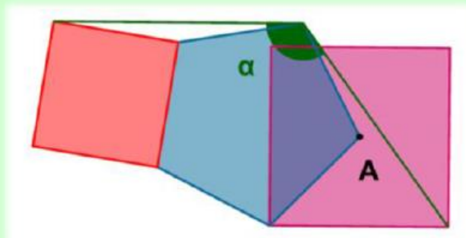
21 Regular decagon. Find reasonably α



22 Regular decagon and regular pentagon. Find reasonably α



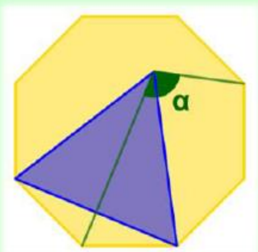
23 **24**



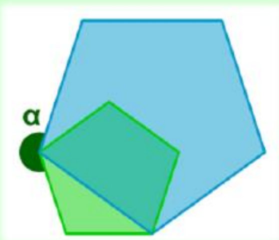
Two squares and a regular pentagon. Vertex A is the centre of the square. Find reasonably α

25

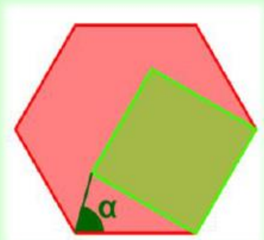
26 Regular octagon and equilateral triangle. Find reasonably α



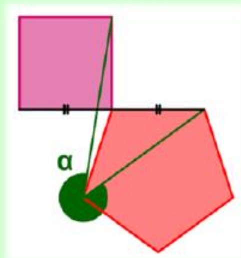
27 Two regular pentagons. Find reasonably α



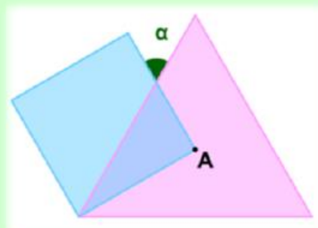
28 Square and regular hexagon. Find reasonably α

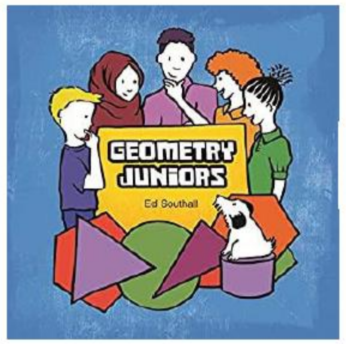


29 Square and regular pentagon. Find reasonably α



30 Equilateral triangle with centroid A and square. Find reasonably α

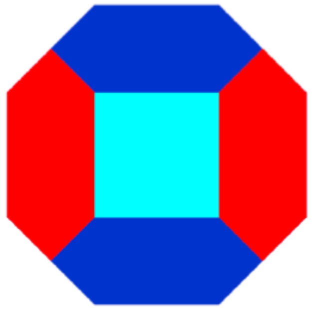




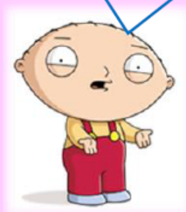
solvemymaths ·

OCTOBER

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	U

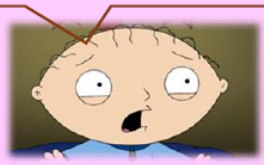

1 Let m and n be odd with m > n, what is the largest integer that divides $m^2 - n^2$?



2

3 Solve in ℕ: $a\log 2 + b\log 3 + c\log 5 = 2022$

4 Let a, b∈ℝ | a > 1, b ≠ 0. If $ab = a^b$ y $\frac{a}{b} = a^{3b}$ Calculate b^{-a}

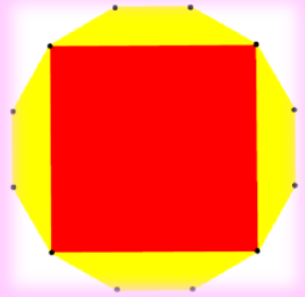


5 In a quiz there are three multiple-choice questions with three answers to each question of which only one is correct. A contestant answers at random, what is the probability that he gets at least two right?

6 Two of the medians of a triangle are ⊥ and measure 8 and 12 cm. Find the area of the triangle

7 What is the area of the regular dodecagon (twelve sides) circumscribed to a square of area 2 dm^2?

8



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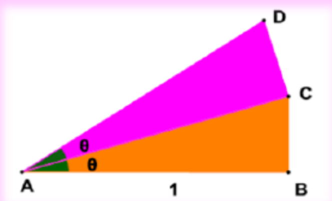
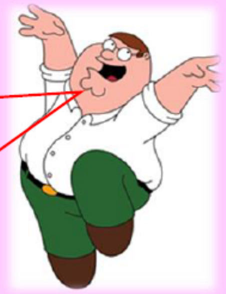
10 In the attached figure find AD as a function of θ.



11

12 Two rays starting from O make an angle of 30° between them. Points A and B are, each one of them, in each of the rays, fulfilling that AB = 1. What is the maximum possible length of OB?



13 Solve in ℕ: $n^4 + 6n < 6n^3 + n^2$

14 Dani has three questions left to answer to finish an exam. Each question has five alternatives of which only one is correct. If she randomly answers these three questions, what is the most likely number of correct answers?

15 How many positive integers less than 2023 verify that any of their digits is zero?

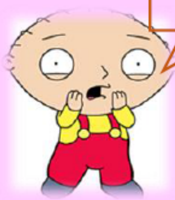
16

17 The key that I put in the lock of my locker has four figures and only two are odd. How many keys meet these requirements?

18 Finding the natural greatest less than 1000, with four different prime divisors.

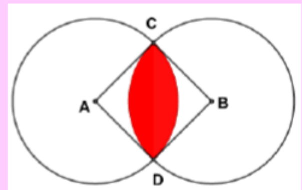
19 I have 18 cards and on each one I have written a 4 or a 5. The sum of all the numbers written is divisible by 17. How many is the 4 written on?

20 Solve in ℝ: $\begin{cases} ab = 26 \\ ac = 128 \\ bc = 52 \end{cases}$



21

22



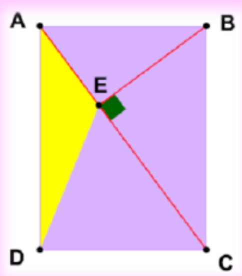
Two circles of radius 2 and centers A and B intersect at C and D. If ACBD is a square, find the area of the red zone

23

24/31

25

In the rectangle in the figure, segment AB measures 3 cm and segment BC measures 4 cm. If E is the foot of the perpendicular from point B to diagonal AC, what is the area of triangle ΔAED?



26 Find the two-digit numbers that satisfy that the product of their digits plus the sum of both coincides with the number.

27 Two of the altitudes of a scalene triangle measure 4 and 12 cm. If the length of the third height is a natural, what is the maximum value of it?

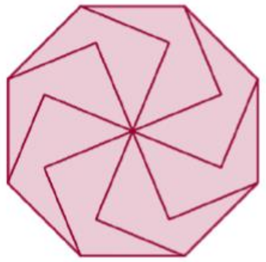
28 A convex polygon has exactly three obtuse angles. What is the maximum number of sides of this polygon?

29 Solve in ℝ: $|x - |2x + 1|| = 3$

30

NOVEMBER

MONDAY | TUESDAY | WEDNESDAY | THURSDAY | FRIDAY | SATURDAY | U



1

2

Dani has sixty marbles and her sisters Laia and Aitana none. Dani decides to take six marbles and give each one three. He believes that by repeating this operation several times, there will come a time when each of the three will have the same number of marbles. Is he right or do I have to do some small adjustment?

3

The average of eight consecutive odd numbers is 42. Calculate the average of all natural odds between the smallest pair and the largest pair.

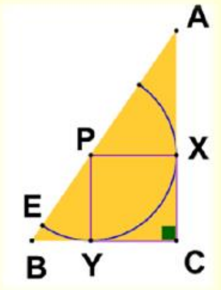
4

5

Laia's grade in a given subject is the arithmetic mean of twelve tests she takes throughout the course. After the first eight checks you have an average grade of 4. What average should you get in the last four checks for the average of all twelve checks to be greater than or equal to five? If in the ninth and tenth checks he draws a six and a five, what average must he draw in the last two checks to draw an average in all the checks greater than or equal to five?

6

7



8

Let ΔABC be a right triangle at C. Let P be a point on AB such that PXCY is a square of side 8 cm. With center in P a circle of radius 8 is drawn that cuts to the segment PB in E. If BE = 2 cm, find the area and the perimeter of ΔABC

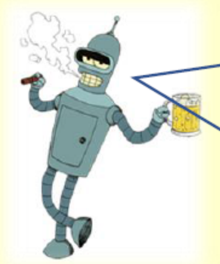
9

Let

$$R = \underbrace{8888..88}_{200} - \underbrace{4343\ldots43}_{140}$$

Is R a perfect square?

10



11

In a whole division the divisor is 49 units greater than the residue and the quotient is 182. If we increase the dividend by 2372 units and keep the divisor unchanged the quotient increases by 28 units and the residue is the maximum allowed. Find the entire first division

12

Calculate how many five-digit or less-numbered captions exist. If they were ordered from lowest to highest, which head would occupy position 195?

13

14

Given a natural n, S_n (P_n) is defined as the sum (product) of the digits of n. Find the natural numbers n such that:

$$P_n \cdot S_n = 3 + P_n$$

15

16

Dani, Laia and Aitana participate in one of the activities of the patron saint festivities of their town: the walk around Benirredrá. A 5.3 km cross country race. Aitana leaves at 10:00 a.m. at a speed of 3 km/h. Ten minutes later Laia leaves, running at a speed of 5 km/h. Twenty minutes later Dani leaves at a speed of 6 km/h. If none of them stop or change their speed, find the order of arrival of the three brothers and the time between their arrival.

17

Given a natural n, S_n (P_n) is defined as the sum (product) of the digits of n. Find the natural n such that:

$$P_n \cdot S_n = 30$$

18

19

Consider the natural number R:

$$R = \underbrace{999\cdots999}_{200} - 1\underbrace{333\cdots333}_{x}$$

Find the values of x that make R a multiple of three.

20

21

			37
			38
			25
33	55	12	

22

Place the first nine prime numbers on the adjoining grid without repeating any so that the total of each line (row or column) is the one indicated on the margin of the grid. Is the solution unique?

23

Dani was born when, Rafael, her father was 32 years old. Now, Dani's age plus his father's exceeds Gregori's by 20 years, which is 52 years. How old is Laia now who was born when the sum of the ages of Dani, Rafael and Gregori was 79 years?

24

25

Leo is a fan of cross country cycling. Three times a week he goes up from the cemetery to the hermitage carrying his bicycle and goes down the same path, but pedaling with his bicycle. If Leo goes up to 12 Km / h:

1.- at what speed you have to go down to have an average speed along the whole journey of 15 km / h.

2.- What is the maximum speed on the whole journey that you can reach?

26

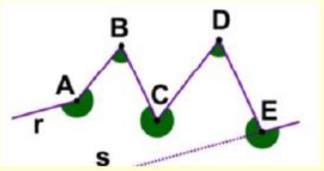
Be given the number:

$$N = \Big(\underbrace{66\cdots66}_{n}\Big)^2 + \Big(\underbrace{33\cdots33}_{n}\Big)^2$$

Is N a multiple of 3? Is it a perfect square?

27

28 (dedicated to Professor Smudge)



If r//s, find the value of the sum of the angles in A, B, C, D and E

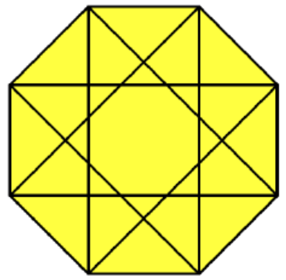
29

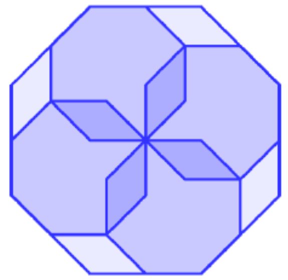
30

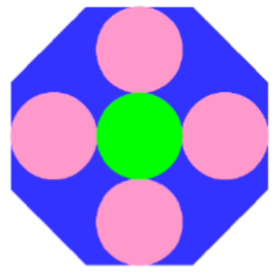
Of three digits, not necessarily different, a, b and c are known:

$$\overline{abc} + \overline{acb} + \overline{bac} + \overline{bca} = 633$$

where $\overline{xyz}$ represents the number with the digit x in the hundreds, the digit y in the tens and the digit z in the units. Find a, b and c



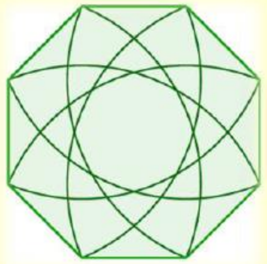


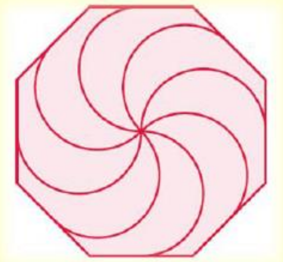


 (OCTAGONS: ALEJANDRO GALLARDO @alegallardo28) AUTHOR: RAFAEL MARTÍNEZ CALAFAT. RETIRED PROFESSOR

DECEMBER

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	U





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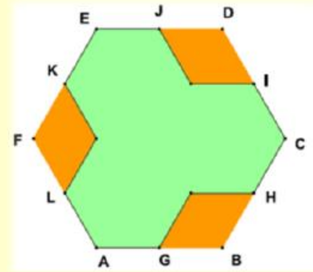


2

Let G, H, I, J, K and L be the midpoints of the sides of the regular hexagon ABCDEF, with area 36 cm². Find the area of the green dodecagon with parallel sides four to four

3

To what exponent must we raise 8 to obtain 16^{21}?

4

5

A ladder 25 m long is leaning against a vertical wall so that the foot of the ladder is 7 m from the wall. If we put it back, now the highest point of the ladder is 4 m lower than before, how far will the foot of the ladder be from the wall now?

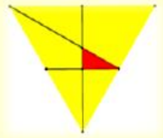
6

7

In the figure there is a regular hexagon ABCDEF of area 180 cm². M and N are the midpoints of AF and ED, respectively.
Also, MO||AB; OP||BC; PN||CD y MO = MF = PN. Find the area of the octagon ABCDNPOM

8

In the equilateral triangle in the figure, we have marked the midpoints of the sides. What fraction of the triangle does the red triangle occupy?



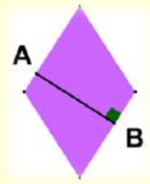
9

10

Dani plays with 5 ABCDE cards to mix them up as follows. Change 1: he takes the card from the centre and puts it first: CABDE. Change 2: he takes the last card and puts it in the middle: CAEBD. Change 3 equals change 1. Change 4 equals change 2, and so on. When making the 2022 change, how have the cards been ordered?

11

12

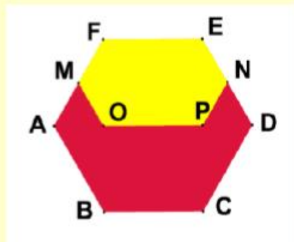


The perimeter and area of the rhombus is 24 cm and 24 cm². Calculate the length of the segment AB

13

Calculate the smallest n∈ℕ such that $2^8 + 2^{11} + 2^n$ is a perfect square

14



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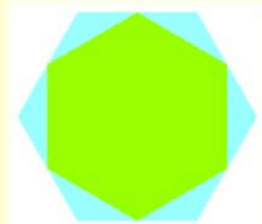
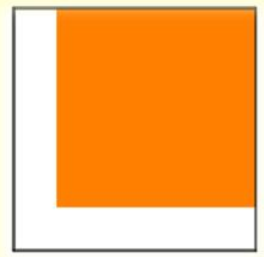


16

We inscribe in a regular hexagon another regular hexagon whose vertices are the midpoints of the sides of the first. What is the ratio between the areas of the hexagons?

17



18

19

The acute triangle ΔABC has the angle at B of 58° and at C of 42°. Also, the bisector at A intersects the opposite side at P. In triangle ΔAPC, the bisector at P intersects the opposite side at D. Find the measure of the angle ∠PDA

20

21

Two runners A and B leave, at the same time, from a city X to another Y, which is 30 km away. Runner A is going 4 km/h less than Runner B. When B reaches Y, he turns around and finds A 6 km from Y. What is the speed of runner A?

22

Dani's father is triple Dani's age. If we add the two figures of the father's age with the two figures of Dani's age, we obtain Dani's age. In addition, the sum of the two digits of the father's age is equal to the sum of the two digits of Dani's age. Calculate the age of both.

23

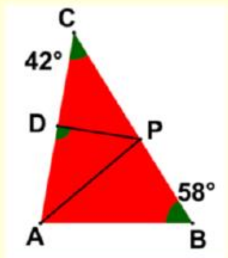
Of the naturals a and b it is known that **a + b** ends in 1 and that $a^2 + b^2$ ends in 3. In what number does $a^{2022} + b^{2022}$?

24

On top of a white square, an orange one whose side measures 2 cm less than the white one has fallen on top, as indicated in the figure. If the surface of the white area of the figure is 36 cm², how many cm is the orange square?

25

26



27

The letters x, y, and z represent nonzero digits. Find the number xyz knowing that the sum is well done.

```
  x x
  y y
+ z z
-----
z y x
```

28

We have 8 cards with the numbers $2^0, 2^1, 2^2, 2^3, 2^4, 2^5, 2^6$ y 2^7. Laia takes a few and Aitana the rest. The sum of those of Laia exceeds the sum of Aitana by 31. How many cards did Laia take?

29

30

Aitana takes 24 minutes to complete a certain task, while her nephew Noa takes 3 hours. If they work together, how long will it take them to do the task 51 times?

31

When a barrel is 30% full, it contains 30 liters less than when it is 70% full. How many liters does the full barrel contain?

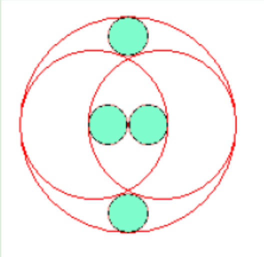
 (OCTAGONS: ALEJANDRO GALLARDO @alegallardo28)

AUTHOR: COLLECTIVE "CONCURSO DE PRIMAVERA" (http://www.concursoprimavera.es/#concurso)

JANUARY

MONDAY | TUESDAY | WEDNESDAY | THURSDAY | FRIDAY | SATURDAY | U

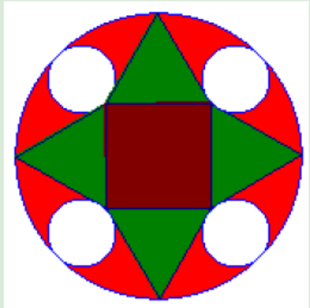
2



3

Two equal median circles and four small circles have been drawn on an exterior circumference of radius R.
Calculate the radius of the circles.
Ishikawa Chiefdom

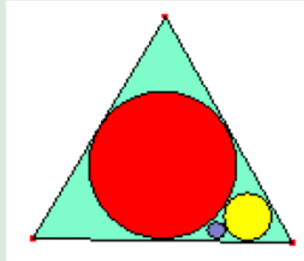
4



5

A circle of radius R contains four equal circles and two other equal circles. Calculate the radius of all the circles.
Fukushima Headquarters

6 Φ, φ DAY



7

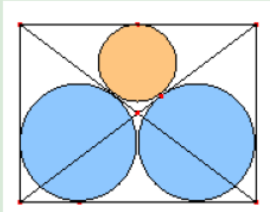
Given an equilateral triangle, a circle of radius r has been inscribed. Another circle is tangent outside the inscription and on two sides of the triangle. A third circle is tangent to one side and tangent outside to the previous two circles. Calculate the radius of the circumferences.
Saitama Headquarters

1/8

9

Two circles of equal radius r are tangent and each of them is tangent to two sides of a rectangle and to a diagonal. Calculate the measure of the sides of the rectangle and the radius of the outer tangent circumference to the previous ones and tangent to the rectangle.
Fukushima Headquarters

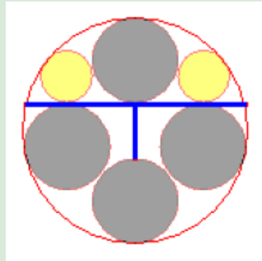
10



11

Inside a circumference of radius R, a square has been drawn, 4 equilateral triangles on the sides of the square and 4 circumferences tangent to the exterior circumference and tangent to the sides of the triangle. Calculate the radius of these 4 circles.
Okayama Headquarters.

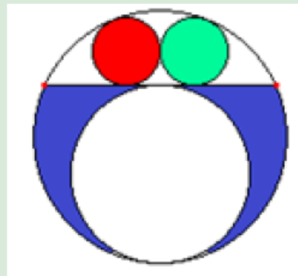
12



13

In the figure there are two small circles of radius r and one large circle of radius s inside a circle. Calculate the diameter of the outer circumference.
Nagasaki Headquarters

14



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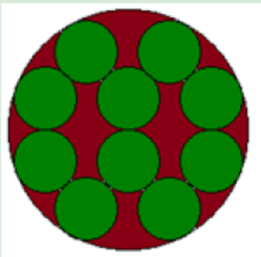
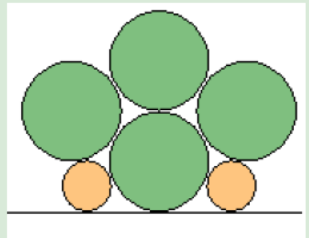
16



17

The figure has six circles tangent three to three. There are four big ones and two small ones. The two small ones and one large one are tangent to a line. Calculate the ratio between the radii of the two types of circles.
Nagano chief.

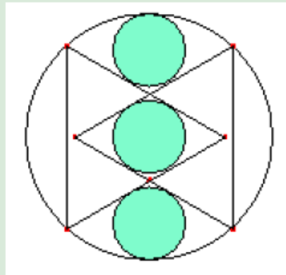
18



19

Two equal equilateral triangles with parallel sides have been drawn in a circle of radius R. At the intersection of the two triangles and on the outside of the two triangles, three circles of equal radius have been drawn. Calculate the radius of the three circles.
Ishikawa Chiefdom

20



21

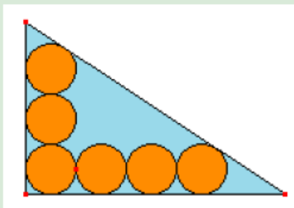
Given a straight line and two equal arcs of radius r tangent to each other and tangent to the straight line, three squares have been drawn. Find the length of the sides of the squares.
Fukushima Headquarters

22

23

Ten equal circles have been inscribed in a circle. Determine the ratio between the radius of a small and the radius of the outer circumference.
Shisouka Headquarters

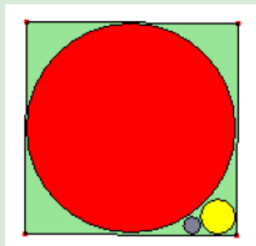
24



25

Six equal and tangent circles of radius r have been inscribed in a right triangle.
Calculate the proportion of the legs.
Calculate the measurement of the legs.
Fukushima Headquarters

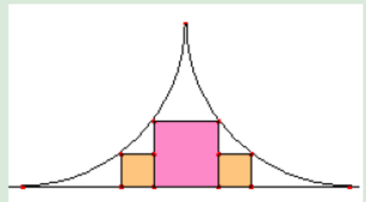
26



27

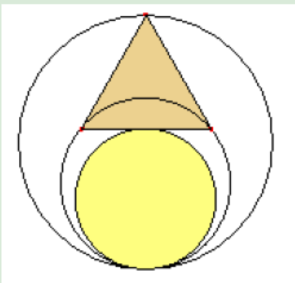
Given a square, a circle of radius r has been inscribed. Another circle is tangent outside the inscription and on two sides of the square. A third circle is tangent to one side and tangent outside to the two previous circles. Calculate the radius of the circles.
Saitama Headquarters

28



29

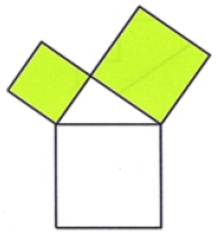
30

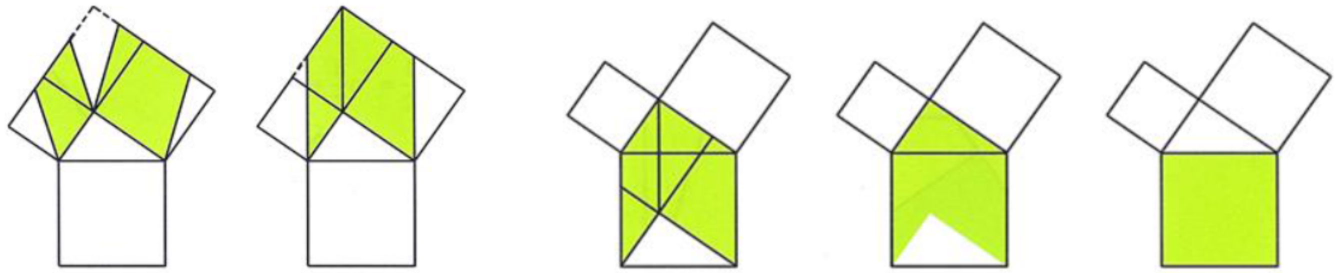


31

Two inner tangent circles at the same tangent point have been inscribed in a circle of radius R. The radius of the small circumference is r. An equilateral triangle is tangent to the small circumference and has two vertices in the middle circumference and the other in the outer circumference. Calculate the radius of the median circumference.
Nagasaki Headquarters

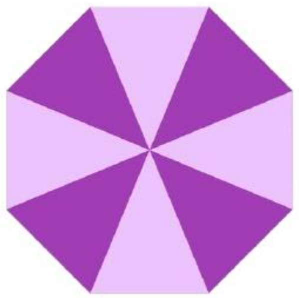
VISUAL PYTHAGORAS





FEBRUARY

MONDAY | TUESDAY | WEDNESDAY | THURSDAY | FRIDAY | SATURDAY | SUNDAY



1

Find the term for the development of the binomial $\left(\frac{3}{4}\sqrt[3]{a^2}+\frac{2}{3}\sqrt{a}\right)^{12}$ which contains a^7

2

For what value of x is the fifth term of the development of $\left(\frac{1}{2\sqrt{x}}-\frac{1}{2}\right)^{10}$ is equal to 105?

3

4

Find the central term of $\left(\sqrt[-7]{\frac{1}{a}}\cdot\sqrt{a}-\sqrt[7]{\frac{a^{-2}}{\sqrt{a}}}\right)^n$, knowing that the coefficient of the fifth term is to the coefficient of the third term as 11 is to 1.

5

6

In the development of the binomial $\left(x^3+\frac{1}{x}\right)^n$ the coefficients of the fourth and eighteenth terms are equal. Find the term where it appears x^4

7 e-day

8

In the expression $\left(\frac{\sqrt[3]{m^2}}{\sqrt[x+1]{m^x}}+m\sqrt[x]{m^{x-2}}\right)^{10}$ find x so that the seventh term is $210m^6$.

9

Find for what value of x, the sum of the second and fourth terms in the expansion of $\left(\sqrt{2^{x+1}}+\frac{1}{\sqrt{2^x}}\right)^m$ is iqual to $\frac{129}{2}\sqrt{2}$, knowing that the sum of the binomial coefficients of the last three terms is equal to 11.

10

11

The sum of all the coefficients of the expansion of the binomial $\left(\sqrt[3]{x}+\sqrt[-2]{\frac{1}{x}}\right)^m$ is 64. Find the term where the exponent of x is $\frac{5}{2}$

12

13

The fourth term of the expansion of the binomial $\left(\frac{\sqrt[3]{5}}{(\sqrt[3]{x})^{10logx}}+x\cdot\sqrt[logx^3]{x}\right)^6$ is 100. Find x.

14

15

Find the value of x in development $\left(\sqrt[4]{x}+\frac{1}{\sqrt{x}}\right)^8$, knowing that the term containing x raised to an exponent that is 5/2 the exponent of the next term, is 144 units larger than the last term mentioned.

16

17

Find the ninth term of a geometric progression whose second term is the complex $\frac{2}{i}$ and the ratio is $2+i$.

18

The systems $\begin{cases}x-y=a\\2y-x=b\end{cases}$ i $\begin{cases}x+2y=c\\x+y=22\end{cases}$ they have the same solutions. Find a, b and c knowing that a, b and c are in geometric progression.

19

20

Let a, b and c be the three-sided lengths of a triangle. We know that a and b are the roots of the polynomial $x^2-(c+6)x+6(c+3)$. Find the largest angle of the triangle.

21

22

In a geometric progression the first term is the coefficient of the sixth term of the expansion of $(x+y)^8$, and the fifth term (of the progression) is the logarithm of the square root of 2187 in base 3. Calculate:

a) the sum of the first ten terms.

b) the sum of the whole series.

23

24

The distance from Pont de Suert to Vilaller is x Km. If we express this distance, successively, in Km, Hm, Dm, m, dm, cm and mm and add all these numbers we get 12,222,221. Find x.

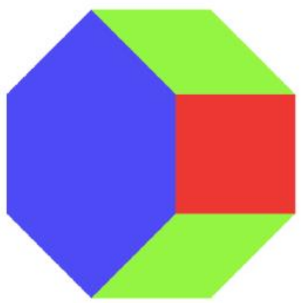
25

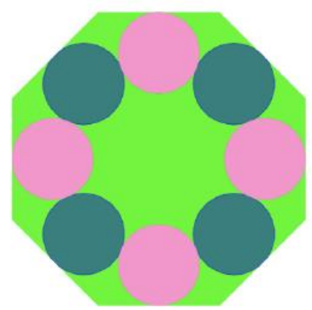
26

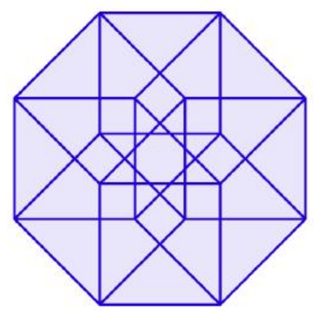
27

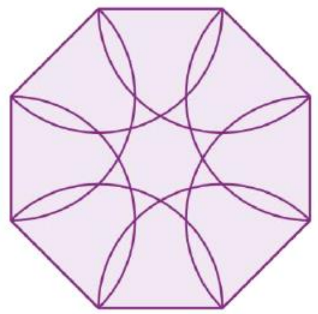
28

Find the coefficient of x^{13} in this expresion $(x^3+1)^2\left(x^2-\frac{2}{x}\right)^8$



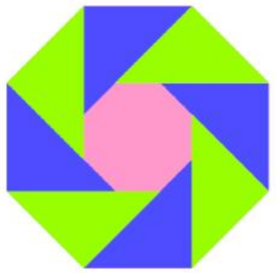
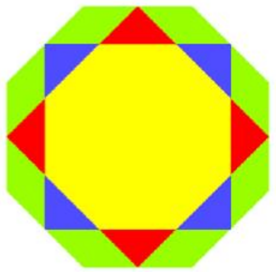






MARCH

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	U

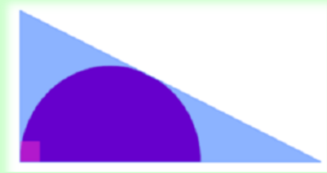



1 Let a and b be natural. If a + b and $a^3 + b^3$ end in 3, what digit does it end in $a^2 + b^2$?

2

3 Rafael has three sons who call him regularly: one every three days, another every four days, and the last one every five days. The last day of 2022 was called by the three sons. How many days in 2023 will he not receive any phone calls from them?

4



In a right triangle of legs 5 and 12 cm, we inscribe a semicircle as in the figure, what is its radius?

5

6 A triangle and a trapezoid have the same area and height. If the base of the triangle is 18 cm, what is the length of the mid-parallel of the trapezoid?

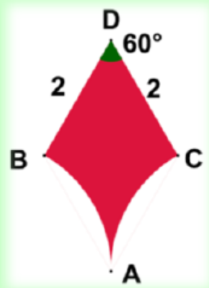
7 If a, b and c are non-zero reals such that a + b + c = 0, find the possible values of:

$$\frac{a}{|a|}+\frac{b}{|b|}+\frac{c}{|c|}+\frac{abc}{|abc|}$$

8 How many scalene triangles are there, with a perimeter less than 13, that have whole number sides?

9 How many trapezoids are there that have an area of 1400 cm^2, a height of 50 cm and a base multiple of 8?

10



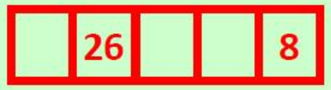
11 In the attached figure DB = DC = 2 cm, ∠BDC = 60°, the length of the arcs BA and CA are one-sixth the length of a circle of radius 2 cm. Find the area of the coloured area.

12

13

14 π day-1

In each box in the figure is written a number so that each of the three exchanges is the arithmetic mean of the two it has attached. Find the numbers written on each box.



15 What is the largest natural n such that 5^n is divisor of 98! + 99! + 100!?

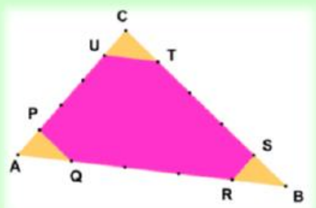
16 The median of a list of five naturals is 1 more than the mode and 1 less than the mean. What is the greatest possible difference between two numbers on the list?

17 We have several natural numbers. The product of the two smallest is 16 and the product of the two largest is 225. Can you find them?

18 In the rectangle of the figure, the lines r and s passing through the vertices A and C are perpendicular to the diagonal BD and divide BD into three segments of length 1 each. Find the area of the rectangle ABCD

19

20



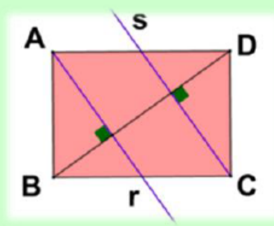
21 We generate a six-digit number N by repeating a three-digit number twice. Is N a multiple of 143?

22 For how many natural numbers n, less than 100, is it verified that n^n is a perfect square?

23

24 Consider 2023 dots, some blue and some green. We assign to each point a fraction whose numerator is the number of points of the other colour and the denominator is the number of points of its colour (including it). What is the sum of the 2023

25



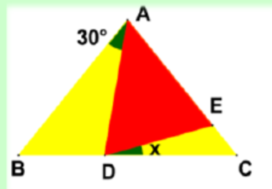
26

27 We have a triangle ΔABC. Each side has been divided into five equal parts (using Thales' procedure). Find the ratio between the area of ΔABC and that of the hexagon PQRSTU

28

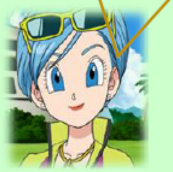
29 Dani, some afternoons, helps deliver goods with his father. One afternoon they plan to travel 210 km. In the end it turns out that they have reached an average speed of 5 km/h more than they had anticipated and they have arrived an hour earlier than expected. What average speed have they reached?

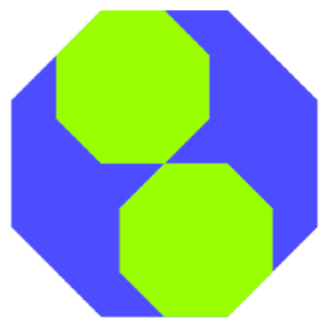
30



In the figure: AB = AC; ∠BAD = 30°; AE = AD. Find x

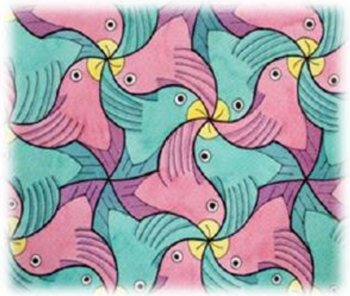
31 What is the probability of obtaining a number that is odd and with all its digits different, if we randomly choose a number between 1000 and 9999?



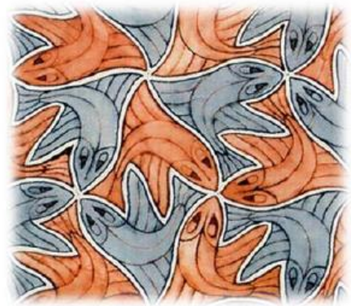


APRIL

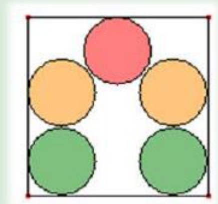
MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	U







1



Five equal circumferences, of radius r are inside a square of side c. Calculate r / c.

Hyogo headquarters

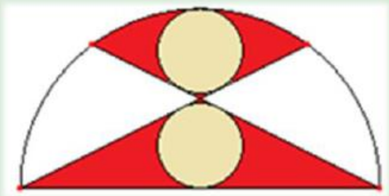
2

3

Two equal circles have been inscribed in a semicircle of radius R (see figure). Calculate the radius of the circumferences.

Aichi Headquarters

4

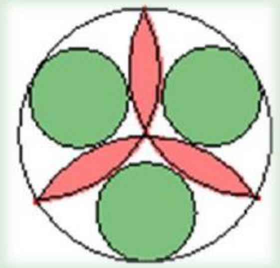


5

A right triangle is inscribed in a circle. Three circles tangent to the previous circle and to the sides of the triangle have been drawn. Let r_1, r_2 be the radii of the circles tangent to the legs. Let R be the radius of the circle tangent to the hypotenuse. Determine the relation between the three radii.

Nagasaki Headquarters

6

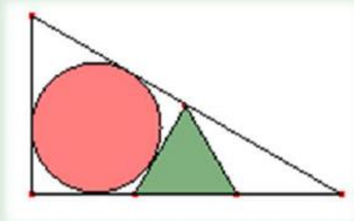


7

8

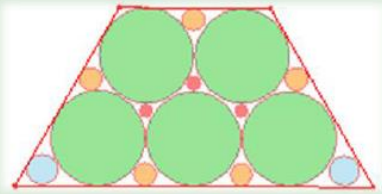
The small leg of the right triangle is c. Calculate the radius of the circumference and the side of the equilateral triangle.

Headquarters Ehime



9

10

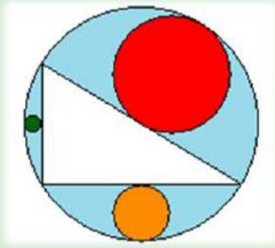


11

The radius of the five green circles tangent to the sides of the trapezoid is r, calculate the radius of the other three types of circles.

Gunma Headquarters

12

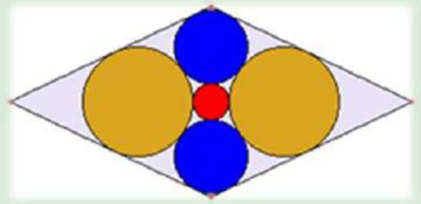


13

In the figure, the three green circles are equal and each one is tangent to an exterior circle and to two arcs. Determine the ratio between the radii of the two types of circles.

Yamagata Headquarters

14



15

Four circles of radii R and r are tangent to the sides of a rhombus. A fifth circle of radius s is tangent to the previous four. Calculate the value of the radius R based on the radii r and s

Nagano Chiefdom

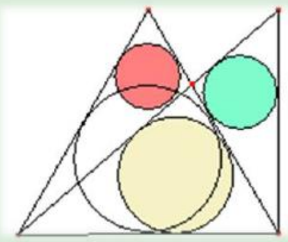
16

17

Calculate the ratio between the sum of the areas of the six equal circles tangent to twelve equal arcs of circumference and the area of the outer circle.

Nagasaki Headquarters

18

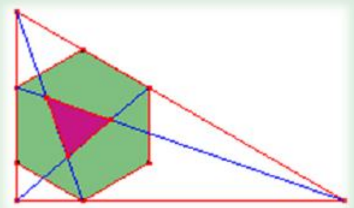


19

In the figure the side of the equilateral triangle is 1. The right triangle has the vertical leg equal to the height of the equilateral triangle. Calculate the radii of the three shaded circles.

Yamagata Headquarters

20

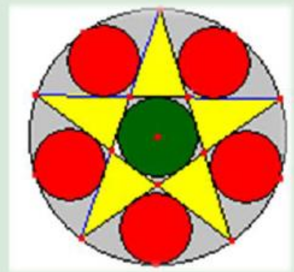


21

A regular hexagon has been inscribed in a right triangle (see figure). Calculate the ratio of proportionality of their areas. The vertices of the right triangle have been joined with the vertices of the regular hexagon. Calculate the ratio between the areas of the triangle and the regular hexagon.

Iwate Headquarters

22



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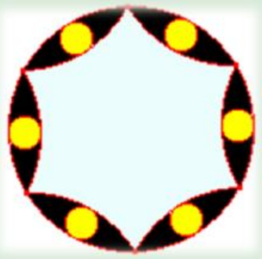

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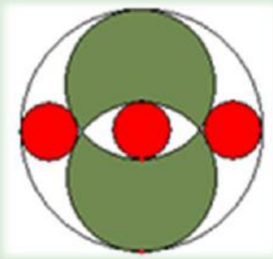
26

Five circles have been drawn in a circumference. The three equal red ones and the other two equal and interior tangents. Calculate the ratio between the radii.

Hyogo Headquarters

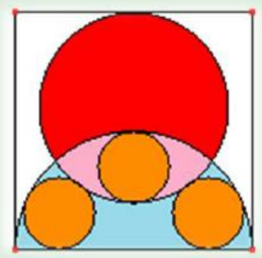


27

Given a square of side 2a we draw a semicircle on the lower side as diameter. We construct a circle of radius R with centre at the perpendicular bisector of the diameter of the semicircle and tangent to the upper side. Three circles of radius r are tangent to the semicircle and to the previous circle. Calculate the ratio between the radii of the two types of circumference.

Iwate Headquarters.

28



29

Consider the regular stellated pentagon and its inscribed circle of radius r. Let the five circles be tangent, of radius s, to the regular star pentagon and to its circumscribed circle. Calculate the ratio: s/r

Nagano Chiefdom

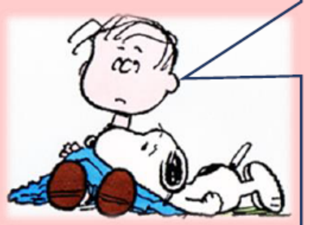
30

MAY

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	U

1

2 Dani mixed concentrated orange juice with 50% water in a pitcher. Aitana drank half of the mixture and to hide it, she filled the pitcher with water. Then came Aitana, who drank half of it and covered it up again by filling the jug with concentrated orange juice. What fraction of the liquid is now water?



3 Aitana has written all the natural numbers from 100 to 199, both inclusive, and Laia has deleted all the digits that are prime numbers. How many digits has Laia erased?

4 Laia picked a basket of oranges from the family garden. She gave half to Dani, 3 oranges to Aitana, 4 to Carles and kept 6 for herself. How many oranges did Laia pick?

5

6 Laia and Aitana jump rope, but at different rates. For every two jumps that Laia makes, Aitana makes seven. The two begin to jump at the same time and when Laia has made a hundred jumps she stops, while Aitana has continued and has made fifty more jumps. How many jumps has Aitana made in total?

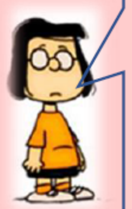
7

8 Obviously, $10 = 5 \cdot 2$. How many two-digit numbers are the product of two single-digit numbers?

9

10 The operations are well done. Different letters represent different numbers. Find the value of each letter

```
    A M O         A M O
        A             A
+ A D A N     + H A D A
  O N D A       M I M O
```



11

12 MATH DAY

Noa is very small. He eats every four hours and poops every eighteen. On Monday at 10 the terrible moment occurred in which he did both things at the same time, which scared her mother, Tània, very much. When will both events coincide again for the first time?

13 We start with the numbers: 1, 2 and 3. The fourth number is the sum of the previous three: 6 and the fifth, the sum of the previous three: 11. If we continue like this, what number will occupy the twentieth position?

14

15

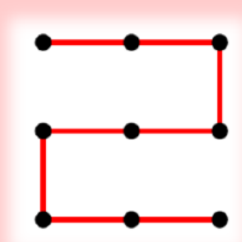
16 Dani plays with her hourglasses that are all thirty minutes long. He flips one and every eight minutes he flips a new clock. Just when he is up forty-seven minutes, how many clocks will still have sand not falling to the top?

17 When Rafa goes on the highway he drives at 120 km/h and when he goes on the national road he goes at 90 km/h. Today he has travelled 560 km in 5 hours. How long is it on the highway?

18 What is the smallest 7-digit number that is a multiple of 73? What about the largest 7-digit multiple of 73?

19

20 Aitana wants to put an unlock key on her mobile that meets these conditions: She has to start at the top left corner, she has to go through the 9 points without ever crossing any previous path and all the lines have to be horizontal or vertical, but they are not worth oblique's. Here we show you a possible key. How many different keys does Aitana have to choose from?



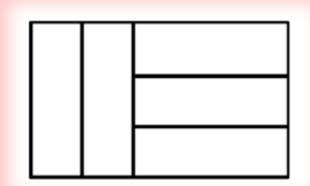
21

22 What is the product of the largest 6-digit multiple of 17 and the smallest 6-digit multiple of 31?

23

24 Dani, Laia and Aitana play with two-digit numbers. Each one writes a couple of them and communicates only one of the two. Dani communicates on the 14, Laia on the 20 and Aitana on the 36. And what are the coincidences, the three products of each pair give the same result! What are the numbers not communicated by Dani, Laia and Aitana?

25



26 Laia and Aitana are designing a flag with five stripes: two vertical and three horizontals, and they can use the colours green, red and blue. If two stripes that touch are prohibited from having the same colour, how many flags are possible?

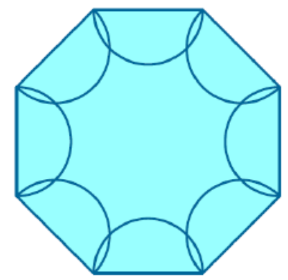
27 Is the number that results from multiplying by 10101 the number formed by two hundred fives a multiple of three?

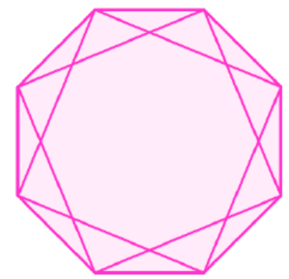
28

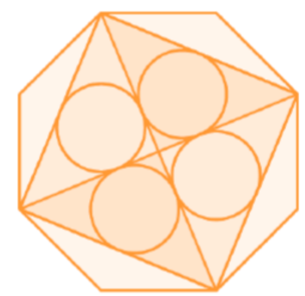
29

30 Laia and Aitana each write a 3-digit number; Ferran and Carles each write a single-digit number and Dani writes a 9. Adding them all together gives 2022. How much do all the digits of the five relatives' numbers add up to?

31 There are 8 red balls, 10 black balls, and 12 blue balls in a bag. We extract balls one by one and without looking inside the bag, how many do we have to extract to ensure that we have 2 red ones?







JUNE

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	U



1 Let ABCDEFG be a regular heptagon with side 1. Prove that:

$$\frac{1}{AC}+\frac{1}{AD}=1$$

2 Five positive integers (not necessarily different) are written on the board and all possible sums of pairs of these numbers are calculated. The only results obtained are 31, 38 and 45 (some of them, several times). What are those 5 numbers?

3 Let S be a set of n elements. Let $p_n(k)$ the number of permutations of the elements of S that leave exactly k elements fixed. Show:

$$\sum_{k=0}^{n} k p_n(k) = n!$$

IMO, 1987, PROBLEM 1

4

5 If α, β and γ ∈ℝ are the angles of a non-right triangle, show that:

$$\tan(\alpha)+\tan(\beta)+\tan(\gamma) = \tan(\alpha)\cdot\tan(\beta)\cdot\tan(\gamma)$$

6 How many odd factors does 20! have?

7 Find all the polynomials P (x) and Q (x) with real coefficients that satisfy:

$$P(Q(x)) = (x-1)\cdot(x-2)\cdot(x-3)\cdot(x-4)$$

8 In each square of a mxn board there is a real number. It is allowed to change all the numbers in a row or column as many times as we want. Show that the sums of the elements in each row and each column can be made non-negative for any initial configuration.

ALL SOVIET UNION COMPETITION 1961. P 7

9 Let a_1, a_2, …. a non-constant PA of real numbers: Suppose there are integers that are prime to each other p, q > 1 for those who a_1^2, a_{p+1}^2 y a_{q+1}^2 are also elements of the same sequence. Prove that all the terms of the sequence are integers.

Indian National Mathematical Olympiad, 2016, problem 6

10 Let x, y and z reals distinct and distinct from 1 and also:

$$\frac{yz-x^2}{1-x}=\frac{xz-y^2}{1-y}$$

prove that both fractions are equal to x + y + z

Olimpiada Iberoamericana, 1985, problema 4

11

12 Consider a convex polygon of area A and perimeter P. Prove that there exists a circle of radius A/P contained inside the polygon

ALL SOVIET UNION COMPETITION 1966, P 7

13 Let n∈ℕ. Prove that the sum of all fractions 1/(pq) where p and q are relatively prime such that 1≤p<q≤n y p+q > n is 1/2

14 On a table there are 100 cards numbered from 1 to 100. 25 of them are chosen at random, what is the probability that the sum of the chosen numbers is even?

15 Prove that if x, y ∈]-1, 1[, then:

$$\left|\frac{x-y}{1-xy}\right| = \frac{|x|+|y|}{1+|xy|}$$

OME, fase local 2004, problema 7

16 Let us consider the set:

S = {1, ½, 1/3, ¼, ⋯⋯ , 1/1000}

We repeat the following process until there is only one element left in S: we choose two numbers x, y∈S and substitute the number x + y + xy. Prove that the last number does not depend on the numbers chosen at each step and calculate it.

17 Find all ways to express 2003 as the sum of the squares of two integers.

OME, fase local 2004, problema 9

18

19 Given a set M of 1985 positive integers, none of which has a prime divisor greater than 26, show that we can find 4 distinct elements of M whose geometric mean is an integer

IMO, 1985, P 4

20 We will say that a circumference is a separator of a set of 5 points in the plane if it passes through 3 of them and the other two, one is inside and the other is outside. Prove that every set of 5 points that does not contain 3 aligned points and 4 concyclic points has exactly 4 separators.

IMO, SHORTLIST GEOMETRY, 1999, P 2

21 Find the value of the real m so that the polynomial

$$x^4-\frac{3\sqrt{2}}{2}x^3+3x^2+mx+2$$

has two real roots, one inverse of the other

22 In a square ABCD a circle is drawn passing through the vertex A and through the midpoints of BC and CD. Determine if the length of the circumference is greater than the perimeter of the square.

23 Let P be an interior point of an equilateral triangle ΔABC such that PA = 5, PB = 7 and PC = 8. Find the length of one side of the ΔABC.

Olimpiada Iberoamericana, 1985, problema 2

24 Let P(x) be a polynomial with integer coefficients such that the equation P(x) = 7 has at least four integer solutions.
Prove that the equation P(x) = 14 has no integer solutions.

25

26 Given k∈ℕ, let A_k the subset of {k+1, k+2,..,2k} formed by the numbers that in base 2 have exactly three ones and is f(k) formed by the numbers that in base 2 have exactly three ones and is A_k. Prove that f (k) = m has at least one solution ∀m∈ℕ. Find the m∈ℕ for which the equation has a unique solution.

27 Let p and q be integers such that:

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots-\frac{1}{1318}+\frac{1}{1319}=\frac{p}{q}$$

Prove that 1979|p

IMO, 1979, PROBLEM 1

28 Consider a triangle whose sides are the sides of a regulars pentagon, hexagon and decagon inscribed in circles of radius 1. Prove that the triangle is a right triangle

29 Suppose that α and β are real that satisfy the equations:

$$\alpha^3-3\alpha^2+5\alpha-17=0$$
$$\beta^3-3\beta^2+5\beta+11=0$$

Calculate α + β

30 Let x, y and z three reals such that:

$$0<x<y<z<\frac{\pi}{2}$$

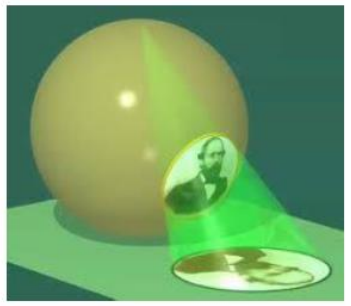
Prove that:

$$\frac{\pi}{2}+2\sin x\cos y+2\sin y\cos z>\sin 2x+\sin 2y+\sin 2z$$

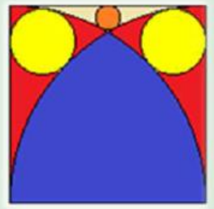
Olimpiada Iberoamericana, 1989, problema 2

JULY

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	U



1 In the figure there is a square, two quadrants and three circumferences, two of them equal. Calculate the ratio of their radii



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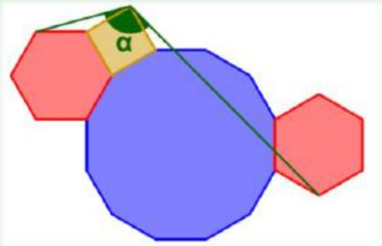
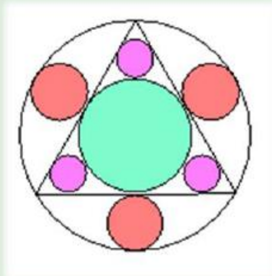


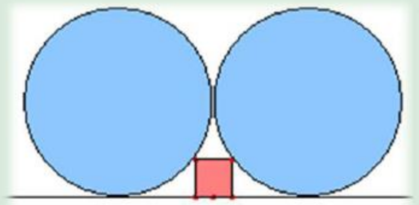
4 Regular dodecagon, two regular hexagons and square. Find reasonably α

5



6 Eight circles are outer tangents two by two and all are outer tangents to one. Calculate the ratio between the radii of the two types of circles. Calculate the ratio between the areas of the sum of the eight blue and the red.

7

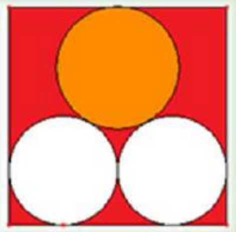


8 Two tangent circles of radius R are tangent to a straight line. Two vertices of a square touch the two circles and the other two vertices are on the line.
Determine the side c of the square in terms of R.
Okayama Headquarters

9

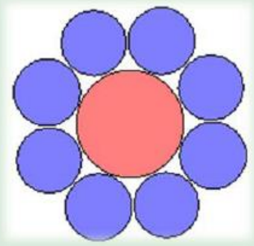
10 Inside a square there are three circumferences, two of them with the same radius, tangent two to two. Calculate the ratio of the radii of the circles.
Aichi Headquarters

11

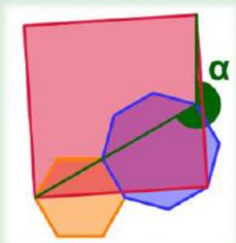


12 An equilateral triangle is inscribed in a circle of radius R. 7 circles have been drawn. One inscribed to the triangle, three external tangents to the triangle and tangents to the first circumference. And, finally, three interior tangents to two sides of the triangle and to the inscribed circle. Calculate the radii of the circles.
Chiba Headquarters

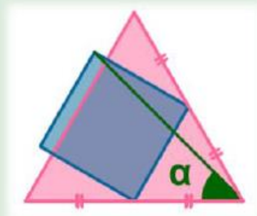
13



14 Regular hexagon, regular octagon and square. Find reasonably α



15 Equilateral triangle and square. Find reasonably α



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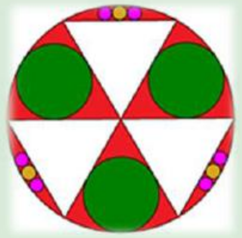
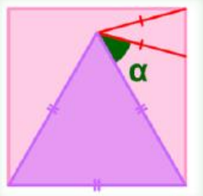
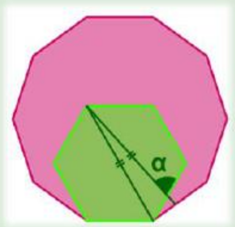


Suwa Nagano Temple.

18 Square and equilateral triangle. Find reasonably α

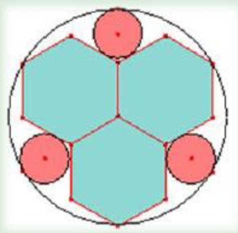


19 Regular decagon and regular hexagon. find reasonably α



20 In a circle of radius R there are three equal regular hexagons and three equal circles, each one tangent to the outer circle and to one side of two hexagons. Calculate the radius of the circles.
Gunma Headquarters. Satimiya Shrine, 1824

21

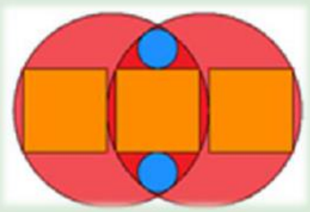


22 π day-2 In the sangaku there is a circle of radius R and 6 circles of the same radius inside it. Three tangents two to two and two of them tangent to a chord. Three lower tangent and aligned. Three of these are interior tangents to the circumference of radius R. Calculate the radius of the 6 circumferences.
Suwa Nagano Temple. 1879

23

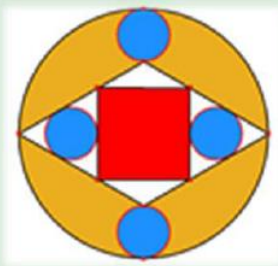
24/31 Outer circle of radius R, three equilateral triangles, three circles tangent to two triangles and to the outer circumference, three circles tangent to the midpoint of the equilateral triangle and tangent to the outer circumference, six circles each tangent to two circles and next to the triangle. Calculate the radii of the three types of circles.

25 Suwa Nagano Temple. 1879



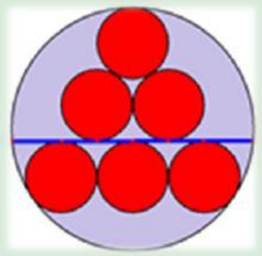
26 Three equal squares have been inscribed in two intersecting circles of equal radius R. The central square is inscribed at the intersection of the two circles. The lateral squares are tangent to the two circles. Two equal circles are tangent to the circles of radii R and to the sides of the central square. Determine the measure of the side of the square and the radius of the tangent circle.

27



28 In the sangaku, a circumference of radius R is shown, a rhombus with a diagonal the diameter of the circumference and the acute angle of 60°. In the rhombus he has inscribed a square Check that the four circles have the same radius.
Suwa Nagano Temple. 1879

29



30